

Complex Power and Mode Coupling in Circular Chirowaveguides

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Abstract—The total power carried by two (or more) modes of lossless circular chirowaveguides is shown to be complex in general, both above and below cutoff. This is explained by the coupling between modes which, in turn, proves the lack of orthogonality, at least in the power sense.

I. INTRODUCTION

THE USE of so-called chirowaveguides in practical applications, as for instance for measurement purposes, calls for an analysis which will often require a mode expansion, e.g. at an air-chiral interface. To this end the dispersion equation of the chirowaveguide must be solved and its eigenmodes determined. For circular chirowaveguides this has been done in detail both above [1], [2] and below cutoff [3]. Commonly, orthogonality relations are used to simplify the expansion. While in empty homogeneous waveguides the complex power serves this need another orthogonality relation has been proposed for chiral waveguides [4]. In this paper, restricting ourselves to lossless circular chirowaveguides, we examine the complex power carried by their modes, and show analytically that these are coupled. From this we infer that an orthogonality in the power sense does not exist.

II. FORMULATION

The complex power P_z carried along the axis of a waveguide, here the z -direction of a cylindrical coordinate system (ρ, ϕ, z) , is given by the integral of the complex Poynting vector over the cross section S of the waveguide. Let us consider two modes u and v . Then

$$P_z = \iint_S [(\mathbf{E}^u \times \mathbf{H}^{u*}) + (\mathbf{E}^v \times \mathbf{H}^{v*}) + (\mathbf{E}^u \times \mathbf{H}^{v*}) + (\mathbf{E}^v \times \mathbf{H}^{u*})] \cdot \hat{z} dS$$

$$= P_z^{u,u} + P_z^{v,v} + P_z^{u,v} + P_z^{v,u}. \quad (1)$$

The asterisk denotes complex conjugation, and \mathbf{E} and \mathbf{H} are the phasors of the electric and magnetic field. In conventional waveguides the first two terms describe the complex power carried by the modes u and v , respectively, while the other two vanish because of orthogonality. We will call these terms modal and coupled power, respectively.

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In the following (1) shall be investigated for the case of a circular waveguide with radius a , perfectly conducting walls and a lossless chiral filling. Following the Born-Drude-Fedorov notation

$$\mathbf{D} = \epsilon(\mathbf{E} + \beta \nabla \times \mathbf{E}) \quad \text{and} \quad \mathbf{B} = \mu(\mathbf{H} + \beta \nabla \times \mathbf{H}) \quad (2)$$

the material is characterized by its permittivity ϵ , its permeability μ , and the chirality parameter β . As was shown in [2] the electromagnetic field can be expressed in terms of left and right circular polarized fields \mathbf{F}_L and \mathbf{F}_R (LCP and RCP), which, in turn, can be expanded into individual modes. For harmonic time dependence $\exp(j\omega t)$ and with the abbreviation $\eta = \sqrt{(\epsilon/\mu)}$ this finally leads to

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}_L + \mathbf{F}_R) = \frac{1}{2} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} d^{mn} \cdot (\mathbf{F}_L^{mn} + r^{mn} \mathbf{F}_R^{mn}),$$

$$\mathbf{H} = \frac{1}{-2j\eta}(\mathbf{F}_L - \mathbf{F}_R) = \frac{1}{-2j\eta} \sum_{m=-\infty}^{\infty} \sum_{n=1}^{\infty} d^{mn} \cdot (\mathbf{F}_L^{mn} - r^{mn} \mathbf{F}_R^{mn}), \quad (3)$$

the transverse components of the individual modes being

$$F_{J\rho}^{mn} = \frac{-je^{-jm\phi} \cdot e^{-jk_z^{mn}z}}{(\alpha_J^{mn})^2} \left[k_z^{mn} \alpha_J^{mn} J'_m(\alpha_J^{mn} \rho) \pm \frac{m\gamma_J}{\rho} J_m(\alpha_J^{mn} \rho) \right],$$

$$F_{J\phi}^{mn} = \frac{-e^{-jm\phi} \cdot e^{-jk_z^{mn}z}}{(\alpha_J^{mn})^2} \left[\pm \gamma_J \alpha_J^{mn} J'_m(\alpha_J^{mn} \rho) + \frac{mk_z^{mn}}{\rho} J_m(\alpha_J^{mn} \rho) \right]. \quad (4)$$

The upper sign refers to LCP ($J = L$) and the lower to RCP ($J = R$). J'_m denotes the derivative of the Bessel function J_m w.r.t. the argument. The azimuthal dependence of the fields is determined by m , while n describes the order of the solution. With $k = \omega\sqrt{(\epsilon\mu)}$, the radial wavenumbers are given by $\alpha_J^{mn} = \sqrt{(\gamma_J^2 - (k_z^{mn})^2)}$, $\gamma_{L(R)} = k/(1-(+)\beta)$ being the wavenumber of the LCP/RCP field, and k_z^{mn} the unknown propagation constant. The latter is a solution of the characteristic equation obtained by applying the boundary conditions. This also leads to the modal field ratios $r^{mn} = -J_m(\alpha_L^{mn}a)/J_m(\alpha_R^{mn}a)$ while the expansion coefficients d^{mn} depend on the source distribution.

The solutions of the characteristic equation have been discussed in detail in [1]–[3]. A typical dispersion curve is depicted in Fig. 1. Above some frequency f_{\min} , k_z is real and splits into two branches which, except for a small backward wave region, correspond to propagation in + and -z-direction. Below f_{\min} , k_z becomes complex. The two solutions, which are now conjugate complex, correspond to a decay in + or -z-direction. The case -m can be obtained from symmetry considerations [3].

With (3) any of the terms (k, l) of P_z in (1) can be written as

$$\begin{aligned} P_z^{k,l} &= \iint_S [\mathbf{E}^k \times \mathbf{H}^{l*}] \cdot \mathbf{z} dS \\ &= \frac{1}{4j\eta} d^k d^{l*} \iint_S [(\mathbf{F}_L^k + r^k \mathbf{F}_R^k) \\ &\quad \times (\mathbf{F}_L^{l*} - r^{l*} \mathbf{F}_R^{l*})] \cdot \mathbf{z} dS \\ &= \frac{1}{4j\eta} d^k d^{l*} \iint_S [(\mathbf{F}_L^k \times \mathbf{F}_L^{l*} - r^k r^{l*} \mathbf{F}_R^k \times \mathbf{F}_R^{l*}) \\ &\quad + (r^k \mathbf{F}_R^k \times \mathbf{F}_L^{l*} - r^{l*} \mathbf{F}_L^k \times \mathbf{F}_R^{l*})] \cdot \mathbf{z} dS, \end{aligned} \quad (5)$$

or, in short form

$$P_z^{k,l} = P_{z1}^{k,l} + P_{z2}^{k,l}. \quad (6)$$

Here, for simplicity of notation, we have used the indices k or l for all mode-characterizing quantities. Using (3) and (4) $P_z^{k,l}$ can also be expressed in terms of the transverse components of $\mathbf{F}_{L,R}$. It is easily shown that the waveguide modes are orthogonal w.r.t their azimuthal order m , i.e. the coupled power is zero. Hence, we will only consider modes with same m in the following. Together with the solutions of the characteristic equation the relations derived in this section completely determine the complex power P_z that will now be analyzed in more detail.

III. ANALYSIS

A. Useful Relations

The detailed analysis of the complex power P_z requires integrals such as in (5) to be investigated. To simplify this task we will indicate some relations for the terms of $P_z^{k,l}$ in (6). An expansion in terms of $\mathbf{F}_{L,R}^{k,l}$ as in (5) immediately leads to

$$\begin{aligned} P_{z1}^{k,l} &= P_{z1}^{l,k*}, \\ P_{z2}^{k,l} &= -P_{z2}^{l,k*}. \end{aligned} \quad (7)$$

In (22) of their paper [4] Engheta and Pelet have derived a relation that verifies mode orthogonality. However, they have assumed real propagation constants. Their approach can be generalized to include the case of complex modes, as they are encountered in chiro-waveguides [3]. Then, as can easily be shown

$$(k_z^k - k_z^{l*}) \iint_S (\mathbf{E}^k \times \mathbf{H}^{l*} + \mathbf{E}^{l*} \times \mathbf{H}^k) \cdot \mathbf{z} dS = 0 \quad (8)$$

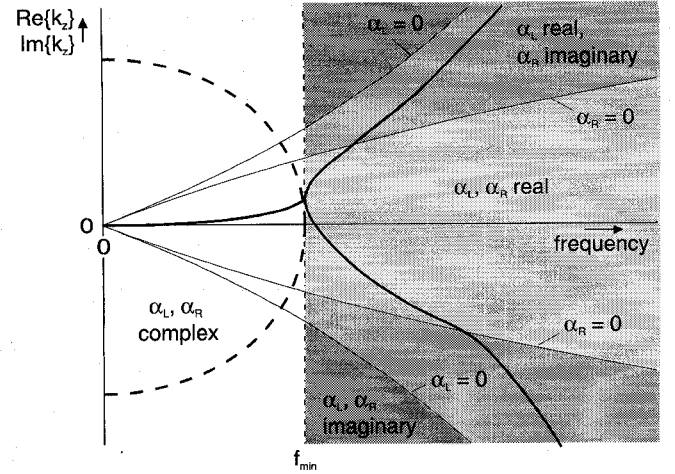


Fig. 1. Real (—) and imaginary (----) part of a typical propagation constant versus frequency in a circular chiro-waveguide.

or, with (5)–(7), equivalently

$$(k_z^k - k_z^{l*}) \cdot P_{z1}^{k,l} = 0 \quad (9)$$

must hold. Equations (8) and (9) are satisfied for $k_z^k = k_z^{l*}$. This situation occurs when $k = l$ and k_z is real, i.e. when a single propagating mode is considered. It is also the case when applying (8) to different complex modes with propagation constants k_z^* and k_z , both solving the characteristic equation [3]. For $k_z^k \neq k_z^{l*}$ the integral, i.e. $P_{z1}^{k,l}$, has to vanish. In particular, this is the case when $k = l$ and k_z is complex (non-real), that is when a single mode below cutoff is considered. Consequently, the biorthogonality relation defined by (8) [5] is not suited for mode expansions because it does not allow to distinguish between identical and different modes. Then (9) yields

$$P_{z1}^{k,l} = 0, \quad (k_z^k \neq k_z^{l*}). \quad (10)$$

B. The Complex Power

The various terms of (1) can now be analyzed. In a first step it is useful to normalize them to the modal expansion coefficients, and to eliminate the z -dependence

$$\tilde{P}_{zi}^{k,l} = \frac{P_{zi}^{k,l}}{d^k \cdot d^{l*}} \cdot e^{j(k_z^k - k_z^{l*})z}, \quad i = 1, 2. \quad (11)$$

Because of their symmetry w.r.t the quantities d^l, d^k, k_z^k and k_z^l the relations (7) and (10) apply as well for $\tilde{P}_{z1}^{k,l}$ and $\tilde{P}_{z2}^{k,l}$. We will now determine whether the corresponding normalized power expression $\tilde{P}_z^{k,l} = \tilde{P}_{z1}^{k,l} + \tilde{P}_{z2}^{k,l}$ is real, imaginary, or complex and, therefore, analyze in detail the integrals defining the power components. To this end one must express the fields in terms of their components as given in (4), and undertake a case study. The following possibilities have to be considered for the propagation constants:

1) $k_z^k = k_z^l$ ($k = l$)

- a) k_z^k real
- b) k_z^k complex

TABLE I
NORMALIZED MODAL AND COUPLED POWER IN A CIRCULAR CHIROWAVEGUIDE

Case	Propagation constants	$\tilde{P}_{z1}^{k,l} =$	$\tilde{P}_{z2}^{k,l} =$	Symmetry
1a)	$k_z^k = k_z^l \in \mathbb{R}$	$\tilde{P}_{z1}^{k,k} \in \mathbb{R}$	0	-
1b)	$k_z^k = k_z^l \in \mathbb{C}$	0	$\tilde{P}_{z2}^{k,k} \in j\mathbb{R}$	-
2a)	$k_z^k \in \mathbb{R}, k_z^l \in \mathbb{R}$	0	$\tilde{P}_{z2}^{k,l} \in \mathbb{R}$	$\tilde{P}_z^{k,l} = -(\tilde{P}_z^{k,l})^*$
2b)	$k_z^k \in \mathbb{R}, k_z^l \in \mathbb{C}$	0	$\tilde{P}_{z2}^{k,l} \in \mathbb{C}$	$\tilde{P}_z^{k,l} = -(\tilde{P}_z^{k,l})^*$
2c)	$k_z^k \in \mathbb{C}, k_z^l \in \mathbb{C}$	0	$\tilde{P}_{z2}^{k,l} \in \mathbb{C}$	$\tilde{P}_z^{k,l} = -(\tilde{P}_z^{k,l})^*$
2d)	$k_z^k \in \mathbb{C}, k_z^l \in \mathbb{C},$	$\tilde{P}_{z1}^{k,l} \in \mathbb{C}$	0	$\tilde{P}_z^{k,l} = \tilde{P}_z^{k,l}$

2) $k_z^k \neq k_z^l (k \neq l)$

- a) k_z^k and k_z^l both real
- b) k_z^k real and k_z^l complex
- c) k_z^k and k_z^l both complex ($k_z^k \neq k_z^{l*}$)
- d) k_z^k and k_z^l both complex ($k_z^k = k_z^{l*}$)

As can be seen from the dispersion diagram in Fig. 1 the radial wavenumbers $\alpha_{R,L}$ can be real, imaginary or complex. The different regions are delimited by the curves $\alpha_R = 0$ and $\alpha_L = 0$ and the frequency f_{\min} , and they are symmetrical w.r.t. the frequency axis. For the analysis these different cases have to be considered separately. As an illustration Case 2a) is treated in detail in the Appendix. For $\tilde{P}_z^{k,l}$ the results have been summarized in Table I. They show that only the classification w.r.t. k_z is important. Using (11) the total complex power P_z carried by two modes u and v can now be calculated from (1). It then reads

$$\begin{aligned}
 P_z = & |d^u|^2 \cdot \tilde{P}_z^{u,u} \cdot e^{-j(k_z^u - k_z^{u*})z} \\
 & + |d^v|^2 \cdot \tilde{P}_z^{v,v} \cdot e^{-j(k_z^v - k_z^{v*})z} \\
 & + d^u \cdot d^{v*} \cdot \tilde{P}_z^{u,v} \cdot e^{-j(k_z^u - k_z^{v*})z} \\
 & + d^{u*} \cdot d^v \cdot \tilde{P}_z^{v,u} \cdot e^{-j(k_z^v - k_z^{u*})z}. \quad (12)
 \end{aligned}$$

In this equation the first two terms that describe the modal power reflect the situation of Case 1 while the last two, the expressions for the coupled power, have to be discussed according to Case 2.

IV. DISCUSSION

In contrast to conventional lossless waveguides the modes in circular chirowaveguides are not independent from one another with the exception of the symmetric ones ($m = 0$). Thus, the power carried by them not only involves modal, but also coupled power. In the following we will examine these different contributions for the cases presented in the previous section.

A. The Modal Power

From the corresponding terms in (12) it is easily seen that, just like in conventional (lossless) waveguides, the power is real above (Case 1a)) and imaginary below cutoff (Case 1b)), although in the latter case k_z is complex in general. This means that the modes either carry active power or behave evanescently, i.e. they store energy.

B. The Coupled Power

Because of the complex expansion coefficients and the exponential z -dependence the coupled power will be complex not only in the Cases 2b), 2c), and 2d) (see Table I), but also in the Case 2a). Thus, formally, each of the terms $P_z^{u,v}$ and $P_z^{v,u}$ represents a transport of complex power. However, these terms cannot exist separately. With the symmetry relations from Table I their sum, i.e. the total coupled power, is always imaginary in the Cases 2a), 2b), and 2c), and real in the Case 2d). In the former cases there is only net energy storage while in the latter we have a net flow of active power.

The coupled power oscillates along the z -axis and is more or less attenuated depending on the kz of the modes. For Case 2a) there is no attenuation at all because both propagation constants are real. The stored energy then has a sinusoidal distribution along the waveguide, being alternately inductive and capacitive. In the Cases 2b) and 2c) the evanescent character of the complex mode(s) involved determines the distribution of the coupled power. Figs. 2(a) and 2(b) illustrate Case 2b), and depict the z -dependence of $P_z^{u,v}$ and $P_z^{v,u}$, respectively. As can be seen the real parts cancel while the imaginary parts are in phase. Again, the total coupled power is alternately inductive and capacitive, but decays exponentially. The numerical values in this example are solutions of the dispersion equation [3], and were chosen so as to get a rather low attenuation. Typically, especially when two complex modes are considered, the exponential decay will be the largely dominating feature.

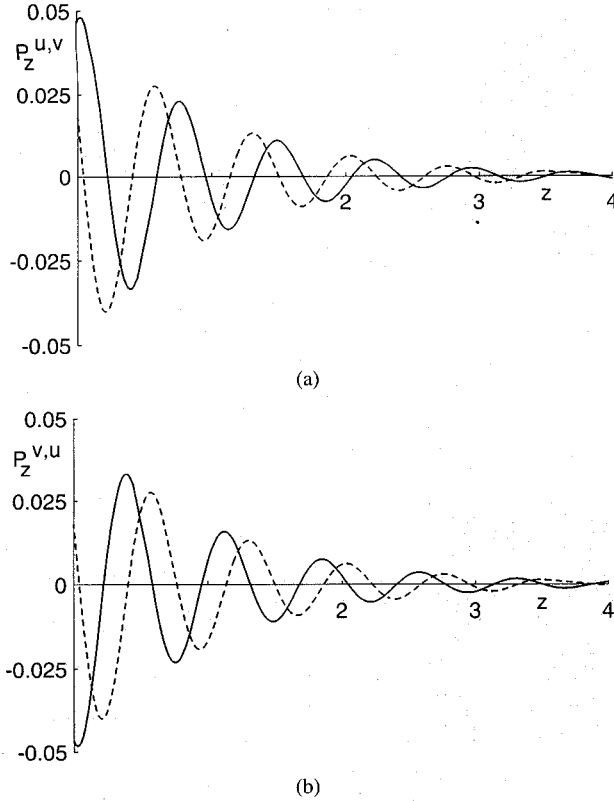


Fig. 2. (a) Active (—) and reactive (-----) part of the coupled power $P_z^{u,v}$ versus z ($m = 1, n^u = 1, n^v = 4, \varepsilon = \varepsilon_0, \mu = \mu_0, \beta/a = 0.05, k \cdot a = 6.2, d_L^u = \exp(-j30^\circ), d_L^v = \exp(-j60^\circ)$). (b) Same as in (a) but for $P_z^{v,u}$.

Finally, the Case 2d) of two modes decaying in opposite directions has to be considered. With $k_z^k = k_z^{l*}$ the exponential z -dependence vanishes. Thus, the total coupled power which is real now remains constant along the waveguide. Physically, this corresponds to a situation where power is transmitted through a waveguide section below cutoff.

C. The Complex Power

As mentioned before the complex power is obtained from (12). If only one mode is considered, then, of course, the complex power reduces to the modal power. For two modes it represents a combination of modal and coupled power which have been discussed above. As an example, let us take Case 2b) with $k_z^u \in \mathbb{R}$ and $k_z^v \in \mathbb{C}$. The modal power is real for mode u (Case 1a)) and imaginary for mode v (Case 1b)). With also the coupled power being imaginary the complex power has an active and a reactive part. The latter will rapidly decay as was explained above for this case. In the same way one can show P_z to be imaginary in the Case 2c), and complex in the Cases 2a) and 2d).

In comparison, in dielectric-loaded waveguides only the complex modes are non-orthogonal and they occur as pairs characterized by $(k_z, -k_z^*)$ and $(-k_z, k_z^*)$. Thus they carry coupled power that is active for the mode combination (k_z, k_z^*) , reactive for $(k_z, -k_z^*)$, or zero for $(k_z, -k_z)$. But, as was shown in [6] and [7], their modal power is zero.

Because the modes couple by pairs the results presented above can, of course, be extended to characterize the complex power carried by any number of modes.

V. CONCLUSION

The complex power carried by the modes of lossless circular chirowaveguides has been analyzed. It was shown that modes with the same azimuthal order are not orthogonal in the power sense. Thus, their complex power is the sum of both modal and coupled power. The total coupled power is imaginary except in the case of two complex modes propagating in opposite directions where it becomes real. As a result the complex power has both active and reactive components.

Some fundamental aspects of wave propagation in lossless circular chirowaveguides were shown to be rather uncommon as compared to the ones of other, more familiar waveguiding structures. In practice, however, lossless chiral media do not exist, and this fact has to be taken into account when analyzing a real case. A generic treatment, though, such as the one presented here, will become more involved then.

APPENDIX

As an example, we will show here that the normalized power expression $\tilde{P}_z^{k,l}$ is real in the Case 2a), where the indices k and l refer to different propagating modes. We then obtain from (5), (10), and (11)

$$\begin{aligned} \tilde{P}_z^{k,l} &= \frac{P_{z2}^{k,l}}{d^k \cdot d^{l*}} e^{j(k_z^k - k_z^{l*})z} \\ &= \frac{1}{4j\eta} \iint_S [r^k (\mathbf{F}_R^k \times \mathbf{F}_L^{l*}) - r^{l*} (\mathbf{F}_L^k \times \mathbf{F}_R^{l*})] \cdot \mathbf{z} dS. \end{aligned} \quad (A1)$$

Inserting the field components from (4) and integrating over ϕ leads to

$$\begin{aligned} \tilde{P}_z^{k,l} &= \frac{\pi}{2j\eta} \int_0^a (r^k \tilde{F}_{R\rho}^k \tilde{F}_{L\phi}^{l*} - r^{l*} \tilde{F}_{R\phi}^k \tilde{F}_{L\rho}^{l*} \\ &\quad - r^{l*} \tilde{F}_{L\rho}^k \tilde{F}_{R\phi}^{l*} + r^k \tilde{F}_{L\phi}^k \tilde{F}_{R\rho}^{l*}) \rho d\rho, \end{aligned} \quad (A2)$$

where \tilde{F} is a shorthand notation of F in (4) that omits the ϕ - and z -dependence. The following cases for the radial wavenumbers have to be distinguished:

- 1) $\alpha_L^k, \alpha_R^k, \alpha_L^l$, and α_R^l are real. Consequently, the modal field ratios r_k and r_l and all \tilde{F}_ϕ are real as well. Due to the imaginary \tilde{F}_ρ all terms of the integrand in (A2) are imaginary, thus $\tilde{P}_z^{k,l}$ is real.
- 2) α_L^k, α_R^k and α_L^l are real, and α_R^l is imaginary. While the ratio r_k is always real then, the other quantities depend on the azimuthal order m :
 - a) m even: r_l and all \tilde{F}_ϕ are real, all \tilde{F}_ρ are imaginary. Consequently, $\tilde{P}_z^{k,l}$ is real.
 - b) m odd: Compared to the even case r_l and $\tilde{F}_{R\phi}^l$ become imaginary, and $\tilde{F}_{R\rho}^l$ real. This results in a real $\tilde{P}_z^{k,l}$.
- 3) α_L^l, α_R^l , and α_L^k are real, and α_R^k is imaginary. This case is symmetric to the previous case, and $\tilde{P}_z^{k,l}$ is also real.
- 4) α_L^k and α_L^l are real, and α_R^k and α_R^l are imaginary. Now, we have:
 - a) m even: r_k and r_l are real. All \tilde{F}_ρ are imaginary and all \tilde{F}_ϕ are real. Again, $\tilde{P}_z^{k,l}$ is real.

- b) m odd: r_k and r_l are imaginary. The field components $\tilde{F}_{Lk\rho}$, $\tilde{F}_{Ll\rho}$, $\tilde{F}_{Rk\phi}$, and $\tilde{F}_{Rl\phi}$ are imaginary and $\tilde{F}_{Rk\rho}$, $\tilde{F}_{Rl\rho}$, $\tilde{F}_{Lk\phi}$, and $\tilde{F}_{Ll\phi}$ are real. Also in this case $\tilde{P}_z^{k,l}$ is real.

As can be seen, $\tilde{P}_z^{k,l}$ is real in all possible cases.

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